Combinatorial Algorithms for Image Segmentation with Elastic Shape Priors

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Corrupted Image Information

Input sequence with 90% uniform noise
Shape Priors for Level Set Segmentation

\[ E_{image}(\phi) \rightarrow \min \]

\[ E_{image}(\phi) + E_{shape}(\phi) \rightarrow \min \]

*Cremers, Osher, Soatto, IJCV '06*
Dynamical Priors for Level Set Segmentation

Statistically synthesized evolution

Tracking with dynamical shape prior

Cremers, PAMI ’06
Dynamical Priors for Level Set Segmentation

Dynamical model of deformation

Joint dynamics of deformation & transformation

Cremers, PAMI ’06
Generalization from a Single Template

template

distortion

occlusion

rotation
Open Challenges

Fewer samples / generalization

Complex shape variability

• Point correspondence

• Part decomposition

Globally optimal solutions
Some Related Work

Elastic Shape Similarity Measures

Maes, Pattern Recognition ‘91
Basri et al., Vision Research ’98
Latecki, Lakämper, PAMI ’00
Manay et al., PAMI ’06
Ling, Jacobs, PAMI ’07
Felzenszwalb, Schwartz, CVPR ’07
Schmidt et al., ICCV ’07

(Elastic) Shape Priors in Segmentation

Coughlan et al., CVIU ’00
Felzenszwalb PAMI ’05
Segmentation, Correspondence and Decomposition into Parts

input silhouette  \rightarrow \text{segmentation}  \rightarrow \text{correspondence}  \rightarrow \text{part decomposition}
Theorem: Any matching of the template to the image corresponds to a cyclic path in the product space of template and image.
Matching Shape to Images: A Combinatorial Solution

Cyclic path \( \Gamma : S^1 \rightarrow \Omega \times S^1 \)
Assigning an Energy / Cost to Each Cycle

\[
\min_{\Gamma=(C,m)} \frac{\int_0^l g(C(s)) ds}{l(C)} + \lambda \frac{\int_0^l |\alpha_c(s) - \alpha_s(m(s))|^2 ds}{l(C)} + \nu \frac{\int_0^l \psi(m'(s)) ds}{l(C)}
\]

- **Data term**
- **Alignment with template**
- **Stretching cost**

\[
g(x) = \frac{1}{1 + |\nabla I(x)|}
\]

Favors strong gradients.

\[
\psi(m') = \begin{cases} 
m' - 1, & \text{if } m' \geq 1 \\
\frac{1}{m'} - 1, & \text{otherwise}
\end{cases}
\]

Favors minimal stretching / shrinking.
Assigning an Energy / Cost to Each Cycle

\[
\min_{\Gamma=(C,m)} \frac{\int_0^l g(C(s)) ds}{l(C)} + \lambda \frac{\int_0^l |\alpha_c(s) - \alpha_s(m(s))|^2 ds}{l(C)} + \nu \frac{\int_0^l \psi(m'(s)) ds}{l(C)}
\]

- **Data term**: Favors strong gradients.
- **Alignment with template**: Favors minimal stretching / shrinking.
- **Stretching cost**: Normalization removes length bias & provides independence of image resolution.

\[
g(x) = \frac{1}{1 + |\nabla I(x)|}
\]

\[
\psi(m') = \begin{cases} 
  m' - 1, & \text{if } m' \geq 1 \\
  \frac{1}{m'} - 1, & \text{otherwise}
\end{cases}
\]

Favors strong gradients.

Favors minimal stretching / shrinking.
Optimization of Ratio Functionals

\[ \frac{\int_{\Gamma} f \, d\Gamma}{\int_{\Gamma} g \, d\Gamma} < \lambda \iff \int_{\Gamma} (f - \lambda g) \, d\Gamma < 0 \]

1. Choose some upper bound \( \lambda \) on the optimal ratio.

2. Find a cycle \( \Gamma \) with negative line energy if existent.

3. If there is one, set \( \lambda \) to the ratio of \( \Gamma \) and goto 2. Otherwise terminate.

Moore-Bellman-Ford algorithm
Optimization of Ratio Functionals

**Lawler 1966**

Minimize ratio in a discrete setting:

\[ E(\Gamma_0) = \sum_{e \in \Gamma_0} \frac{f(e)}{g(e)} = \lambda_1 \]

Iteratively find negative cycles \( \Gamma_k \) in a graph with weights

\[ w_k(e) = f(e) - \lambda_k g(e), \quad k = 1, 2, \ldots \]

where \( \lambda_k \) is the ratio associated with the kth cycle.

Upon convergence we have:

\[ \sum_{e \in \Gamma} \left( f(e) - \lambda_\star g(e) \right) \geq 0 \quad \forall \Gamma \]

which implies:

\[ \lambda_\star < \frac{\sum f(e)}{\sum g(e)} \quad \forall \Gamma \]

\( \lambda_\star = \text{minimal ratio} \)
Matching Shape to Images: A Combinatorial Solution

Stretching / shrinking

Invariance to global rotation
Matching Shape to Images: A Combinatorial Solution

Low contrast tracking

Tracking transparent objects
Matching Shape to Images: A Combinatorial Solution

Schoenemann & Cremers CVPR ’08:
Real-time performance by parallel GPU implementation & more.
Tangent angles differ by local rotation of parts.

How to determine a decomposition into deformable parts?

Decomposition, correspondence & segmentation are coupled.
Segmentation, Correspondence Finding and Decomposition into Deformable Parts

Path in 3D Space $\Gamma : S^1 \rightarrow \Omega \times S^1$
Segmentation, Correspondence Finding and Decomposition into Deformable Parts

Path in 4D Space \( \Gamma : S^1 \rightarrow \Omega \times S^1 \times S^1 \)

- pixel
- reparametr.
- local angle
Segmentation, Correspondence Finding and Decomposition into Deformable Parts

Path in 4D Space $\Gamma : \mathbb{S}^1 \rightarrow \Omega \times \mathbb{S}^1 \times \mathbb{S}^1$

- pixel $C$
- reparametr. $m$
- local angle $\alpha$

$$
\min_{\Gamma = (C,m,\alpha)} \frac{\int_0^l g ds}{l(C)} + \frac{\int_0^l |\alpha_C(s) - \alpha_s(m(s)) - \alpha(s)|^2 ds}{l(C)} + \frac{\int_0^l \Psi(m') ds}{l(C)} + \frac{\int_0^l |\alpha'| ds}{l(C)}
$$

Angle regularity favors piecewise constant angles.
Segmentation, Correspondence Finding and Decomposition into Deformable Parts

input silhouette  \rightarrow \text{segmentation} \rightarrow \text{deformation} \rightarrow \text{part decomposition}
Segmentation, Correspondence Finding and Decomposition into Deformable Parts

input silhouette → segmentation → correspondence → part decomposition
Segmentation, Correspondence Finding and Decomposition into Deformable Parts
Summary

Matching as a shortest cyclic path

Segmentation & point correspondence

Optimal tracking in near realtime

Segmentation & part decomposition