Combinatorial Algorithms for Image Segmentation with Elastic Shape Priors

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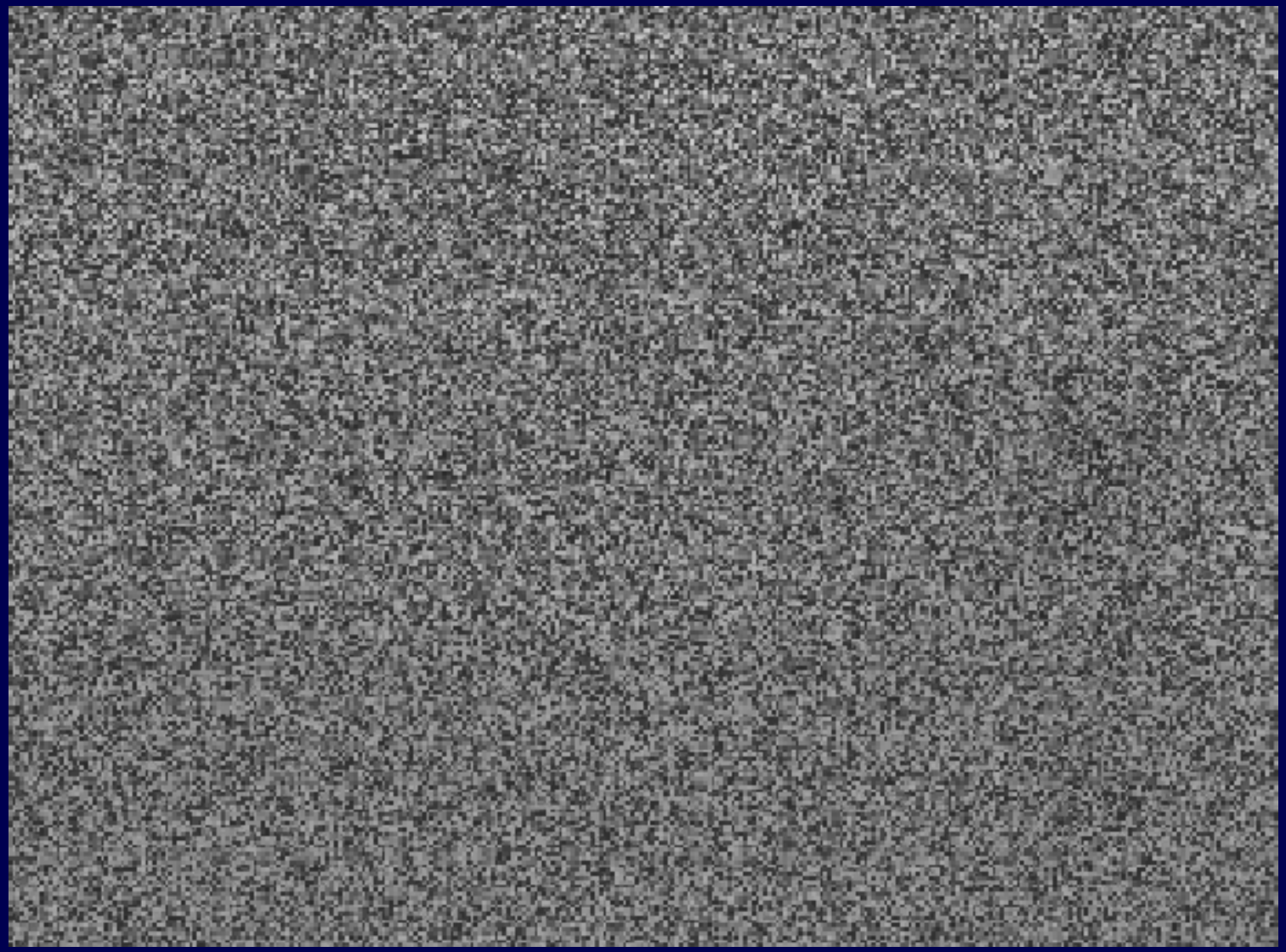
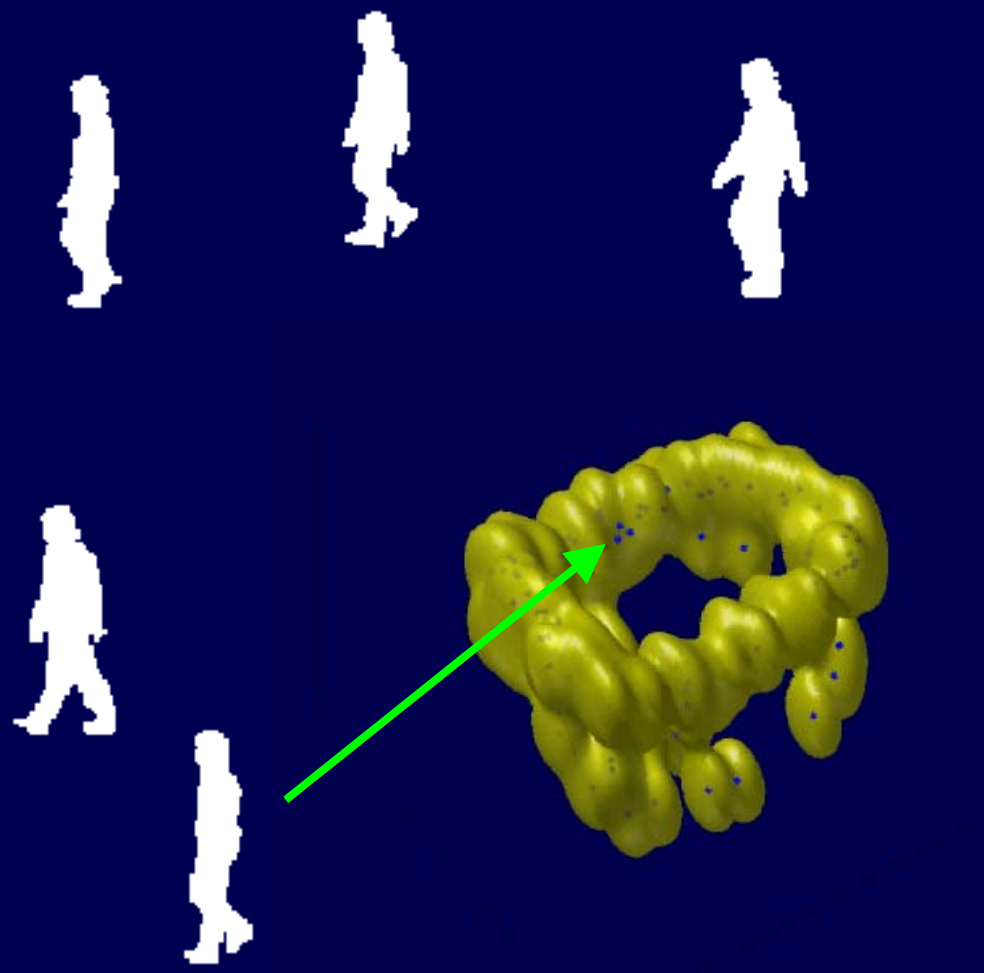
Marseille, October 2008

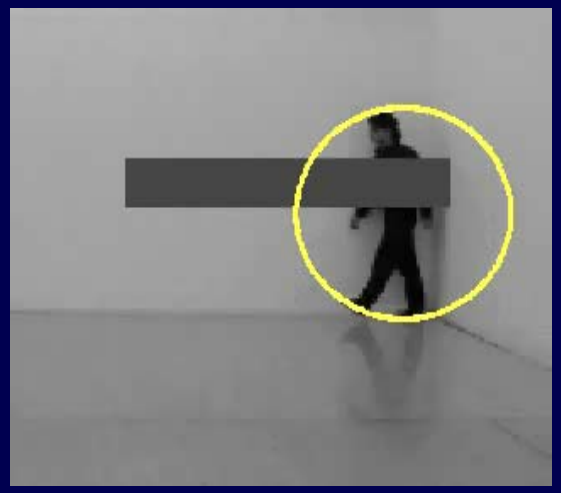
Corrupted Image Information



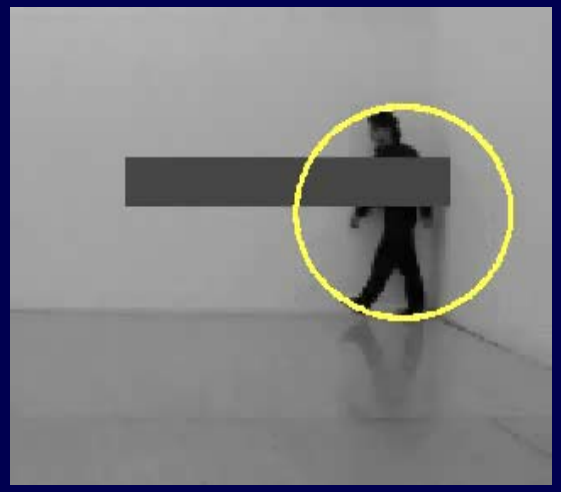
Input sequence with 90% uniform noise

Shape Priors for Level Set Segmentation





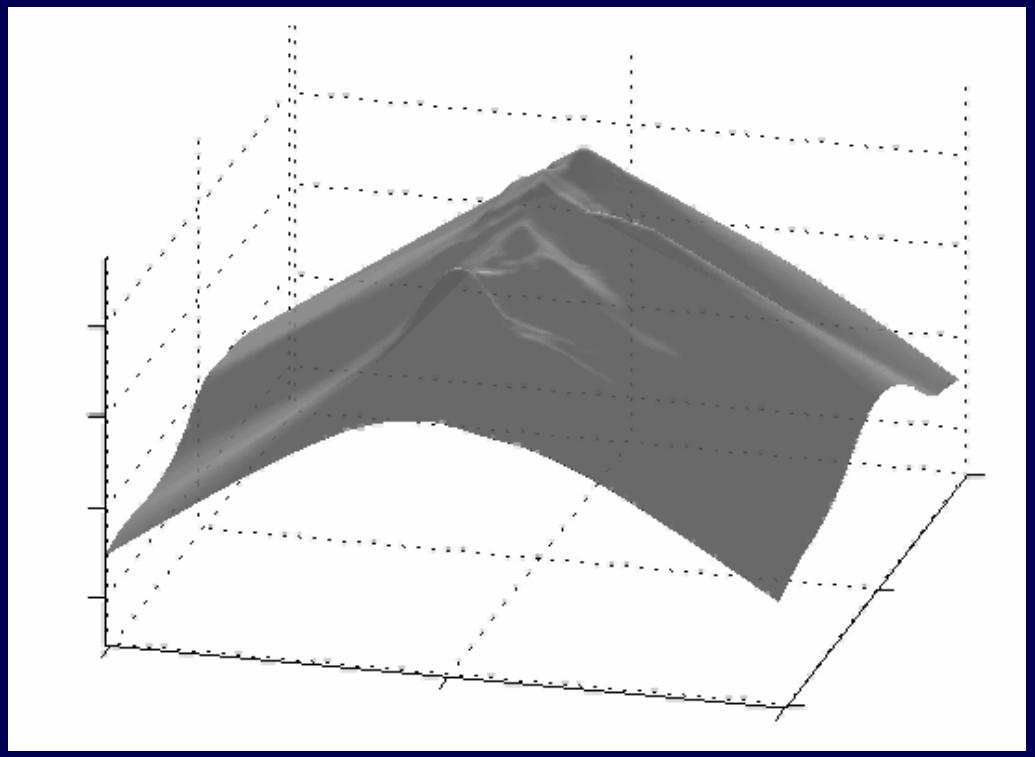
$E_{image}(\phi) \rightarrow \min$



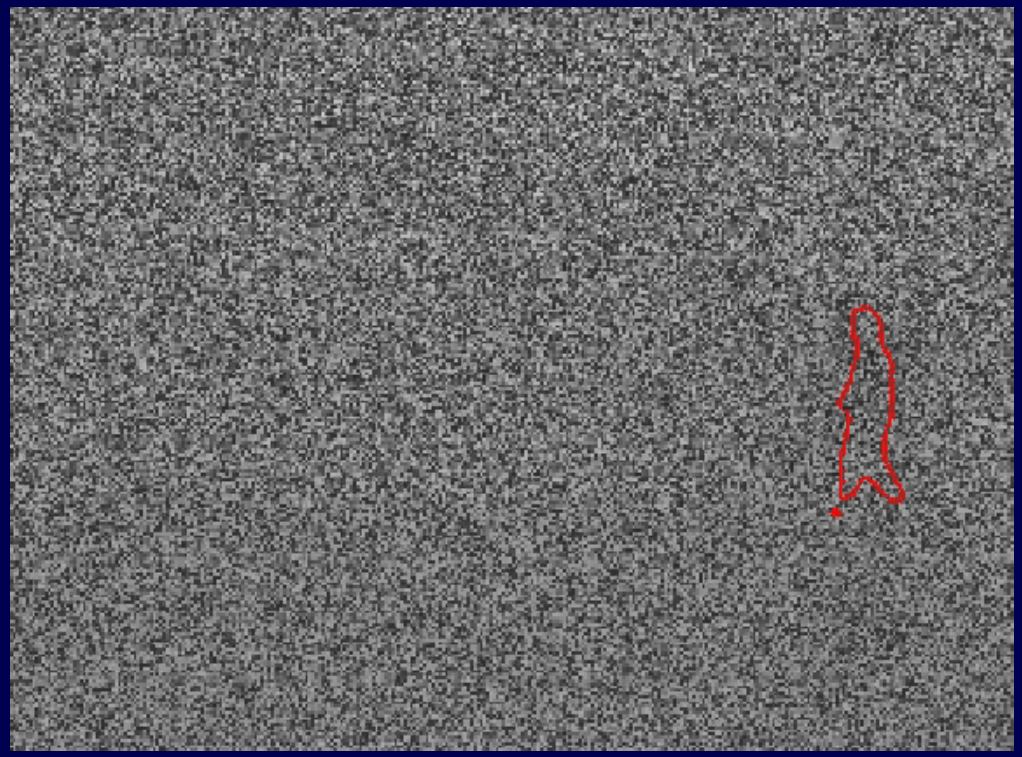
$E_{image}(\phi) + E_{shape}(\phi) \rightarrow \min$

Cremers, Osher, Soatto, IJCV '06

Dynamical Priors for Level Set Segmentation



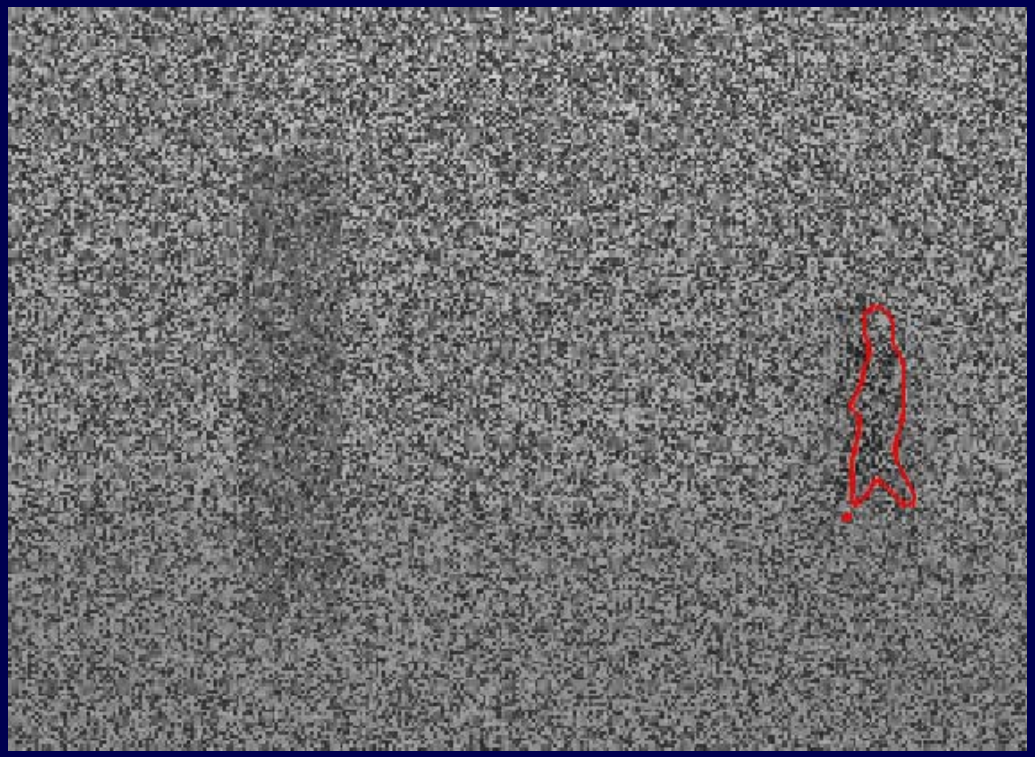
Statistically synthesized evolution



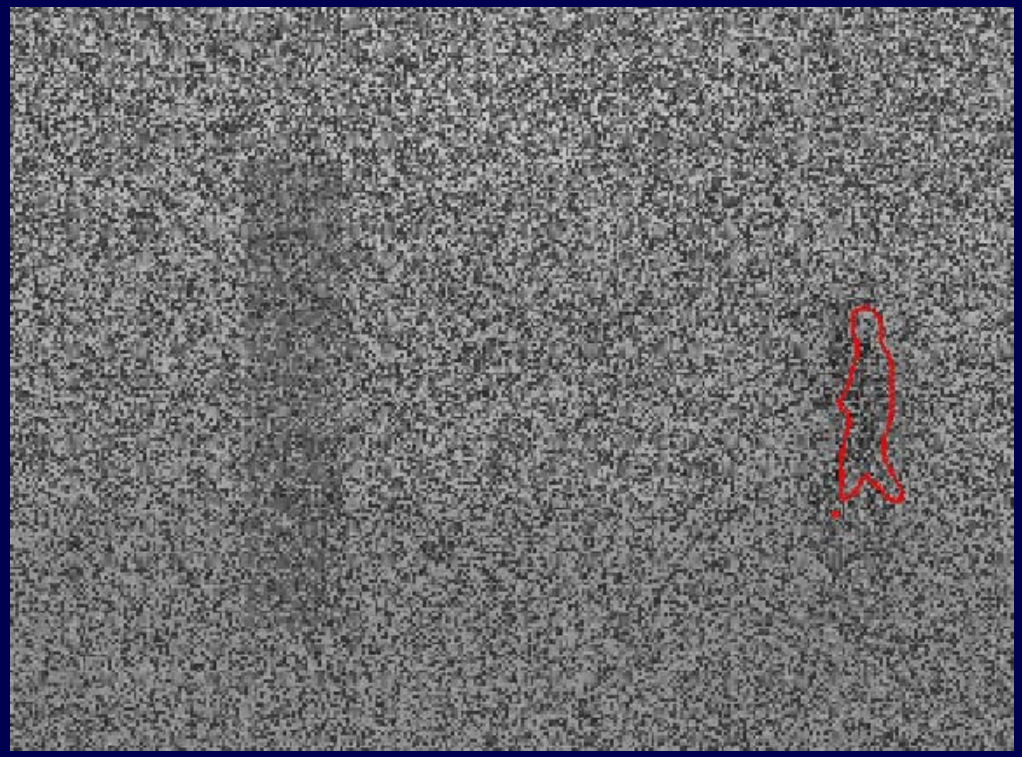
Tracking with dynamical shape prior

Cremers, PAMI '06

Dynamical Priors for Level Set Segmentation



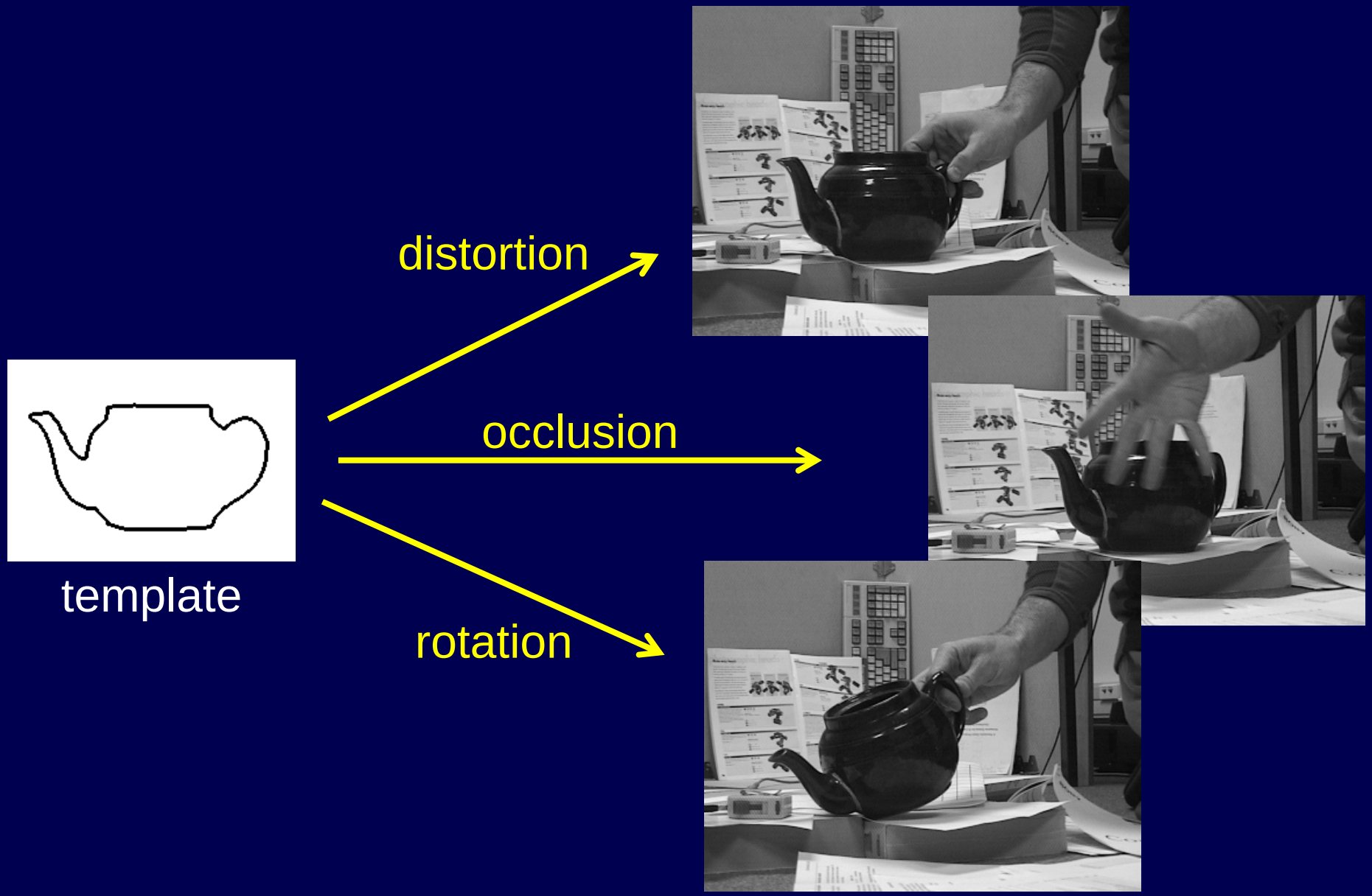
Dynamical model of deformation



Joint dynamics of deformation & transformation

Cremers, PAMI '06

Generalization from a Single Template



Open Challenges

Fewer samples / generalization

Complex shape variability

- Point correspondence
- Part decomposition

Globally optimal solutions

Some Related Work

Elastic Shape Similarity Measures

Maes, Pattern Recognition '91

Basri et al., Vision Research '98

Latecki, Lakämper, PAMI '00

Manay et al., PAMI '06

Ling, Jacobs, PAMI '07

Felzenszwalb, Schwartz, CVPR '07

Schmidt et al., ICCV '07

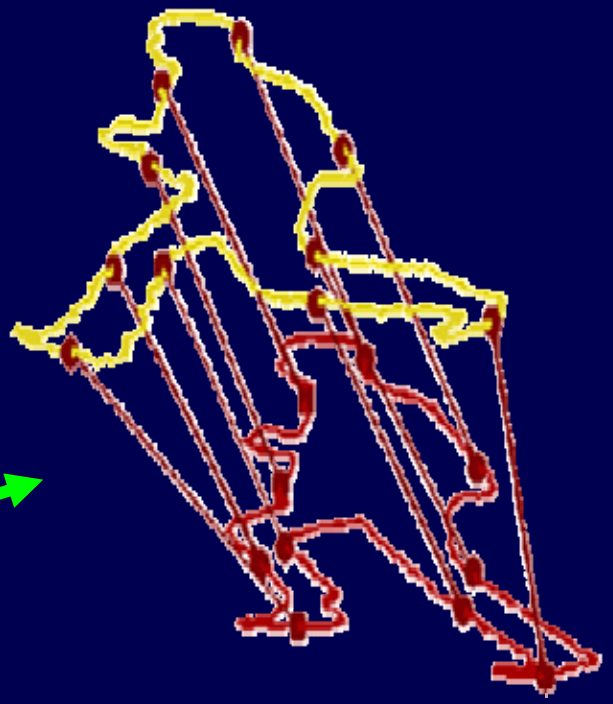
(Elastic) Shape Priors in Segmentation

Coughlan et al., CVIU '00

Felzenszwalb PAMI '05

Segmentation, Correspondence and Decomposition into Parts

segmentation

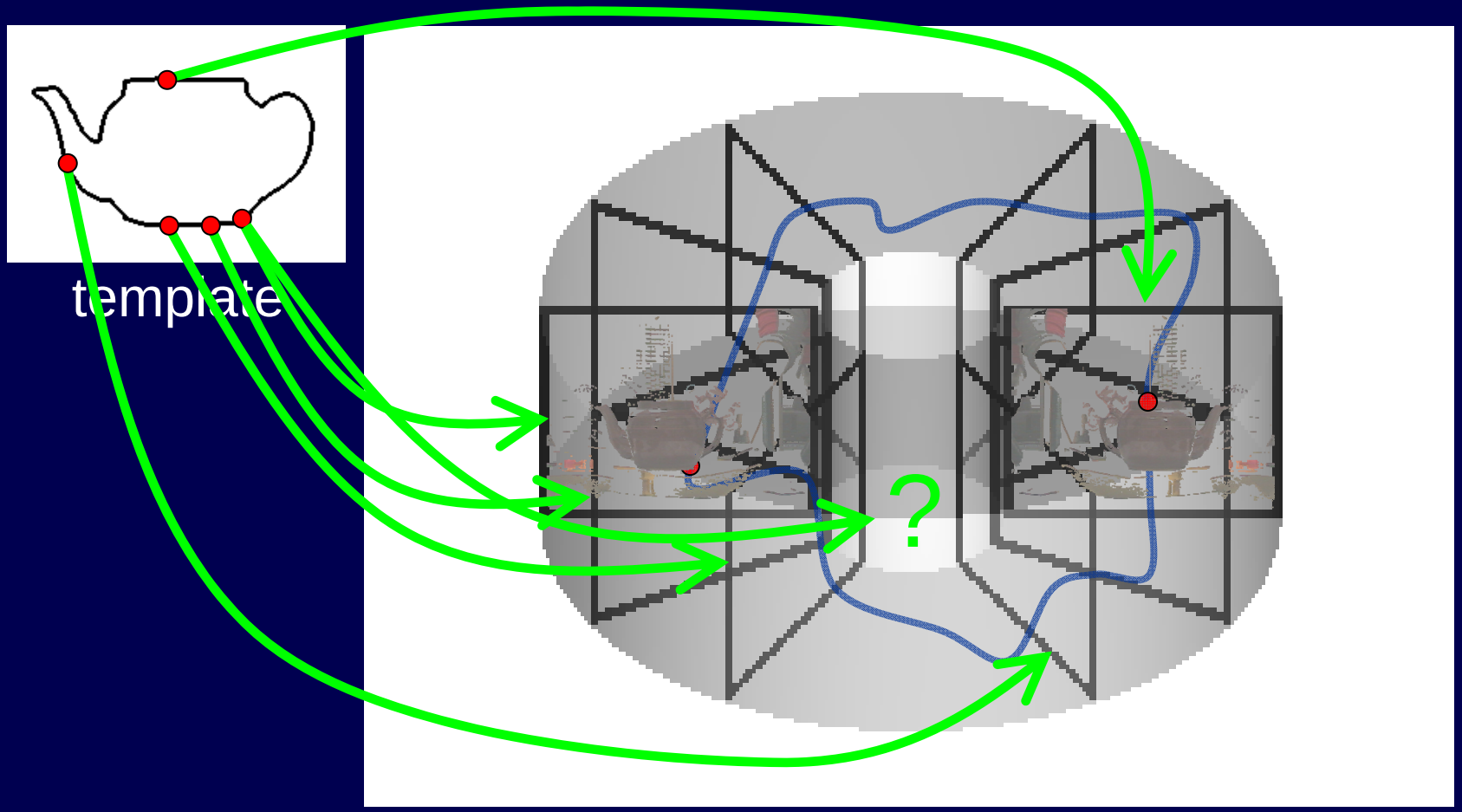


correspondence

input silhouette

part decomposition

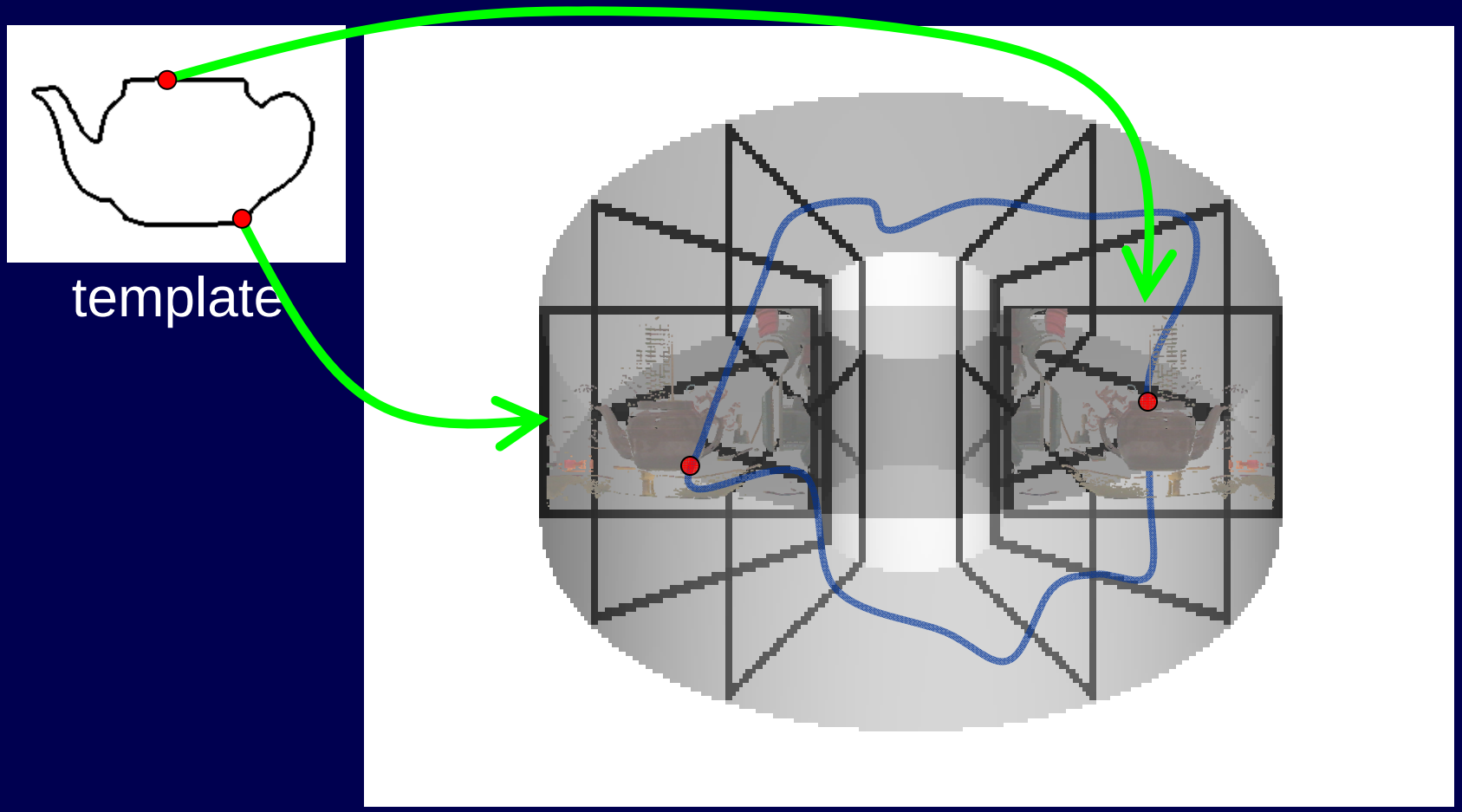
Matching Shape to Images: A Combinatorial Solution



Schoenemann, Cremers ICCV 2007

Theorem: Any matching of the template to the image corresponds to a cyclic path in the product space of template and image.

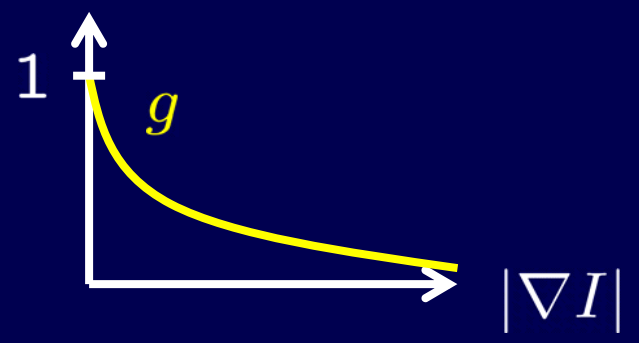
Matching Shape to Images: A Combinatorial Solution



Cyclic path $\Gamma : \mathbb{S}^1 \rightarrow \Omega \times \mathbb{S}^1$

Assigning an Energy / Cost to Each Cycle

$$\min_{\Gamma=(C,m)} \underbrace{\frac{\int_0^l g(C(s))ds}{l(C)}}_{\text{Data term}} + \lambda \underbrace{\frac{\int_0^l |\alpha_c(s)-\alpha_s(m(s))|^2 ds}{l(C)}}_{\text{Alignment with template}} + \nu \underbrace{\frac{\int_0^l \Psi(m'(s))ds}{l(C)}}_{\text{Stretching cost}}$$



$$g(x) = \frac{1}{1+|\nabla I(x)|}$$

Favors strong gradients.

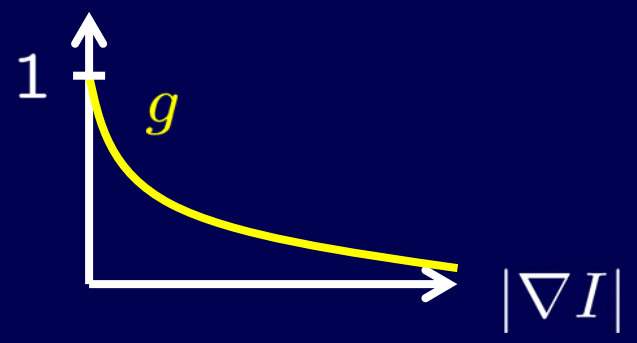
$$\Psi(m') = \begin{cases} m'-1, & \text{if } m' \geq 1 \\ \frac{1}{m'}-1, & \text{otherwise} \end{cases}$$

Favors minimal stretching / shrinking.

Assigning an Energy / Cost to Each Cycle

$$\min_{\Gamma=(C,m)} \underbrace{\frac{\int_0^l g(C(s))ds}{l(C)}}_{\text{Data term}} + \lambda \underbrace{\frac{\int_0^l |\alpha_c(s)-\alpha_s(m(s))|^2 ds}{l(C)}}_{\text{Alignment with template}} + \nu \underbrace{\frac{\int_0^l \Psi(m'(s))ds}{l(C)}}_{\text{Stretching cost}}$$

Normalization removes length bias & provides independence of image resolution.



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Optimization of Ratio Functionals

Lawler 1966

$$\frac{\int_\Gamma f \, d\Gamma}{\int_\Gamma g \, d\Gamma} < \lambda \;\Leftrightarrow\; \int_\Gamma \Big(f - \lambda g \Big) \, d\Gamma < 0$$

1. Choose some upper bound λ on the optimal ratio.
2. Find a cycle Γ with negative line energy if existent.
3. If there is one, set λ to the ratio of Γ and goto 2. Otherwise terminate.

Moore-Bellman-Ford algorithm

Optimization of Ratio Functionals

Lawler 1966

Minimize ratio in a discrete setting:

$$E(\Gamma_0) = \frac{\sum_{e \in \Gamma_0} f(e)}{\sum_{e \in \Gamma_0} g(e)} = \lambda_1$$

Iteratively find negative cycles Γ_k in a graph with weights

$$w_k(e) = f(e) - \lambda_k g(e), \quad k = 1, 2, \ldots$$

where λ_k is the ratio associated with the kth cycle.

Upon convergence we have: $\sum_{e \in \Gamma} \Big(f(e) - \lambda_\star g(e) \Big) \geq 0 \quad \forall \Gamma$

which implies:

$$\boxed{\lambda_\star \leq \frac{\sum f(e)}{\sum g(e)} \quad \forall \Gamma}$$

$\lambda_\star =$ minimal ratio

Matching Shape to Images: A Combinatorial Solution

Stretching / shrinking

Invariance to global rotation

Matching Shape to Images: A Combinatorial Solution

Low contrast tracking

Tracking transparent objects

Matching Shape to Images: A Combinatorial Solution

3 fps

25 fps

Schoenemann & Cremers CVPR '08:

Real-time performance by parallel GPU implementation & more.

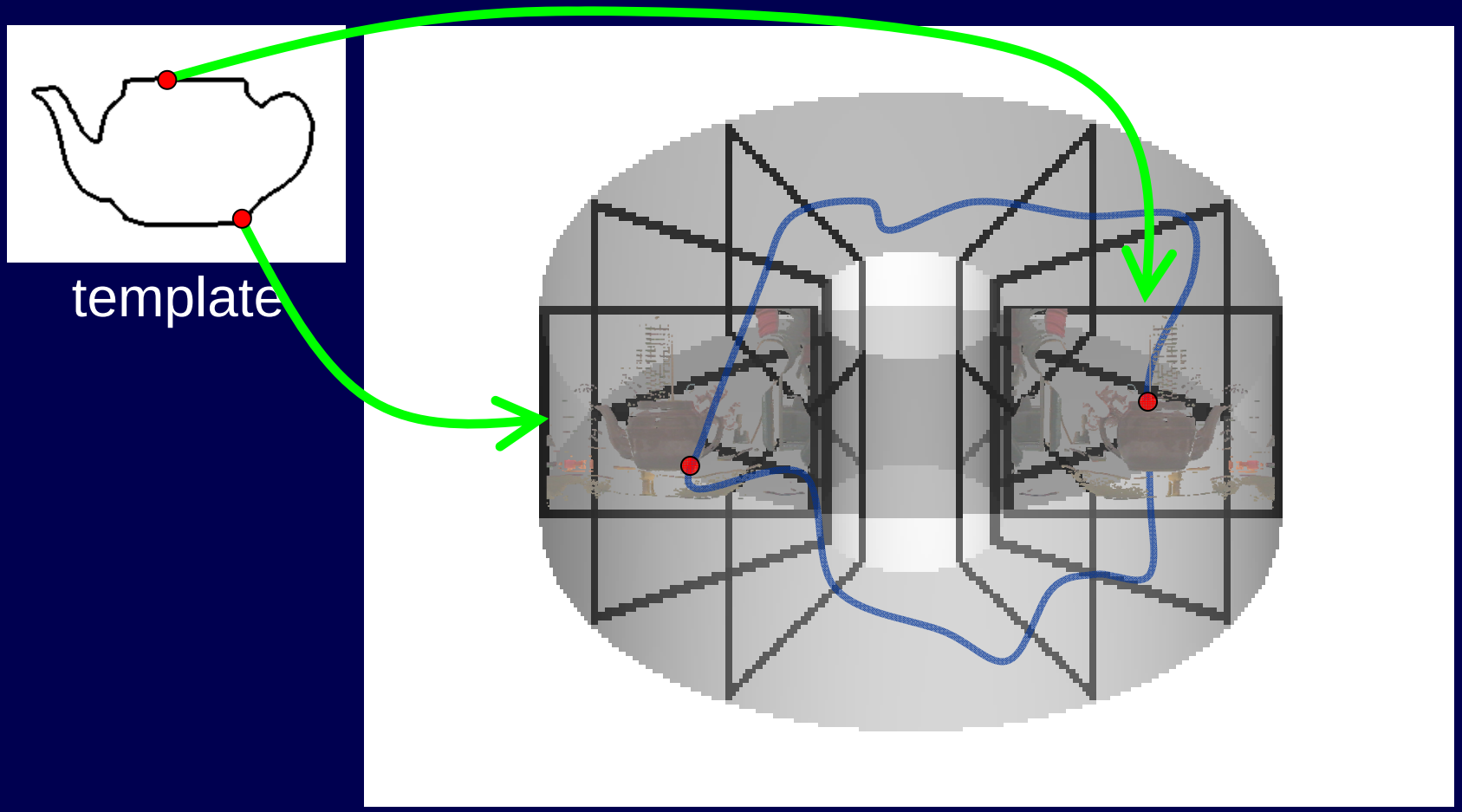
How to Handle Substantial Deformations?

Tangent angles differ by local rotation of parts.

How to determine a decomposition into deformable parts?

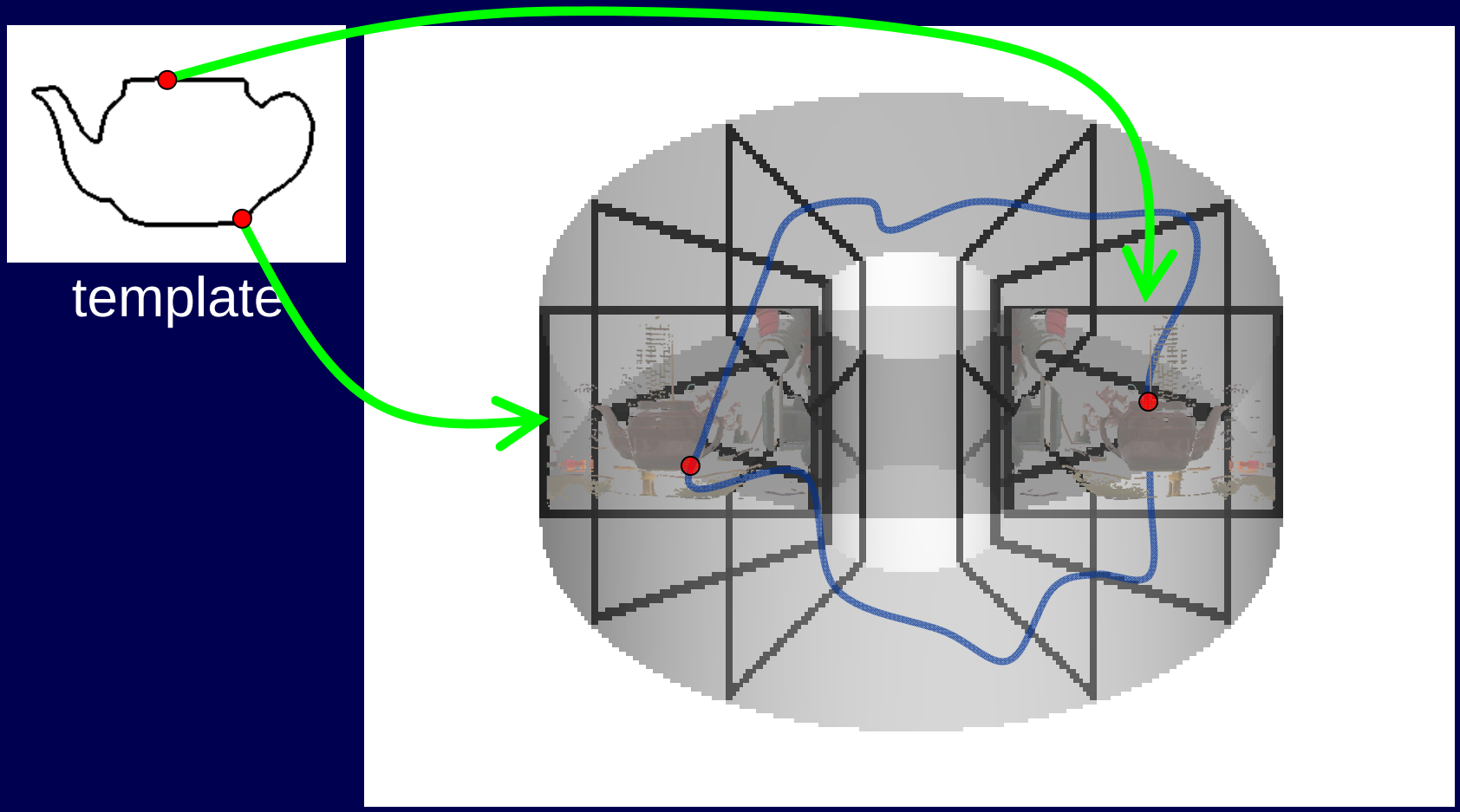
Decompostion, correspondence & segmentation are coupled.

Segmentation, Correspondence Finding and Decomposition into Deformable Parts



Path in 3D Space $\Gamma : \mathbb{S}^1 \rightarrow \Omega \times \mathbb{S}^1$

Segmentation, Correspondence Finding and Decomposition into Deformable Parts



Path in 4D Space $\Gamma : \mathbb{S}^1 \rightarrow \underbrace{\Omega}_{\text{pixel}} \times \underbrace{\mathbb{S}^1}_{\text{reparametr.}} \times \underbrace{\mathbb{S}^1}_{\text{local angle}}$

Segmentation, Correspondence Finding and Decomposition into Deformable Parts

Path in 4D Space $\Gamma : \mathbb{S}^1 \longrightarrow \Omega \times \mathbb{S}^1 \times \mathbb{S}^1$

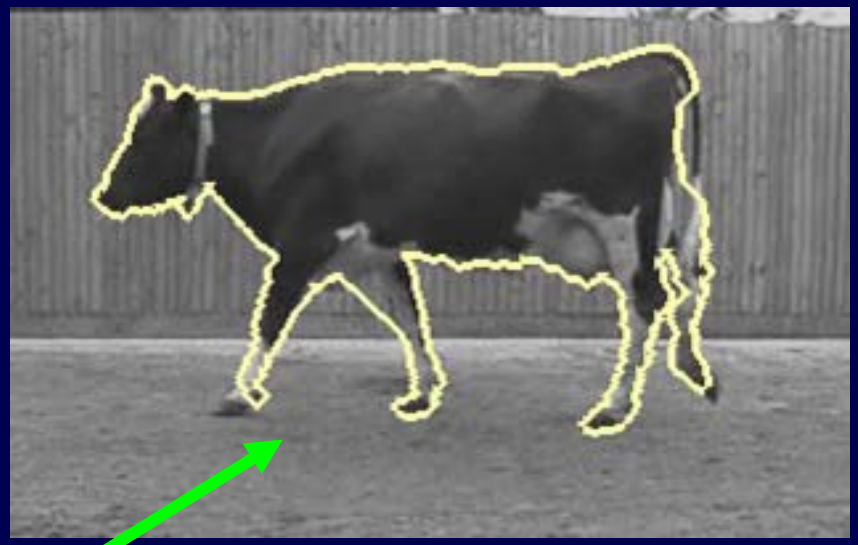
pixel C reparametr. m local angle α

$$\min_{\Gamma=(C,m,\alpha)} \frac{\int_0^l g ds}{l(C)} + \lambda \frac{\int_0^l |\alpha_c(s) - \alpha_s(m(s)) - \alpha(s)|^2 ds}{l(C)} + \nu \frac{\int_0^l \Psi(m') ds}{l(C)}$$

$$+ \frac{\int_0^l |\alpha'| ds}{l(C)}$$

Angle regularity favors piecewise constant angles.

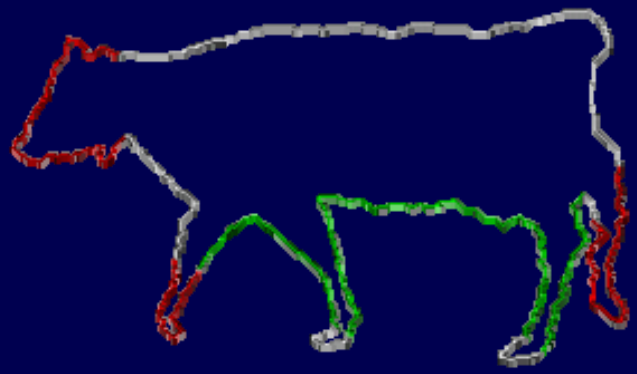
Segmentation, Correspondence Finding and Decomposition into Deformable Parts



segmentation

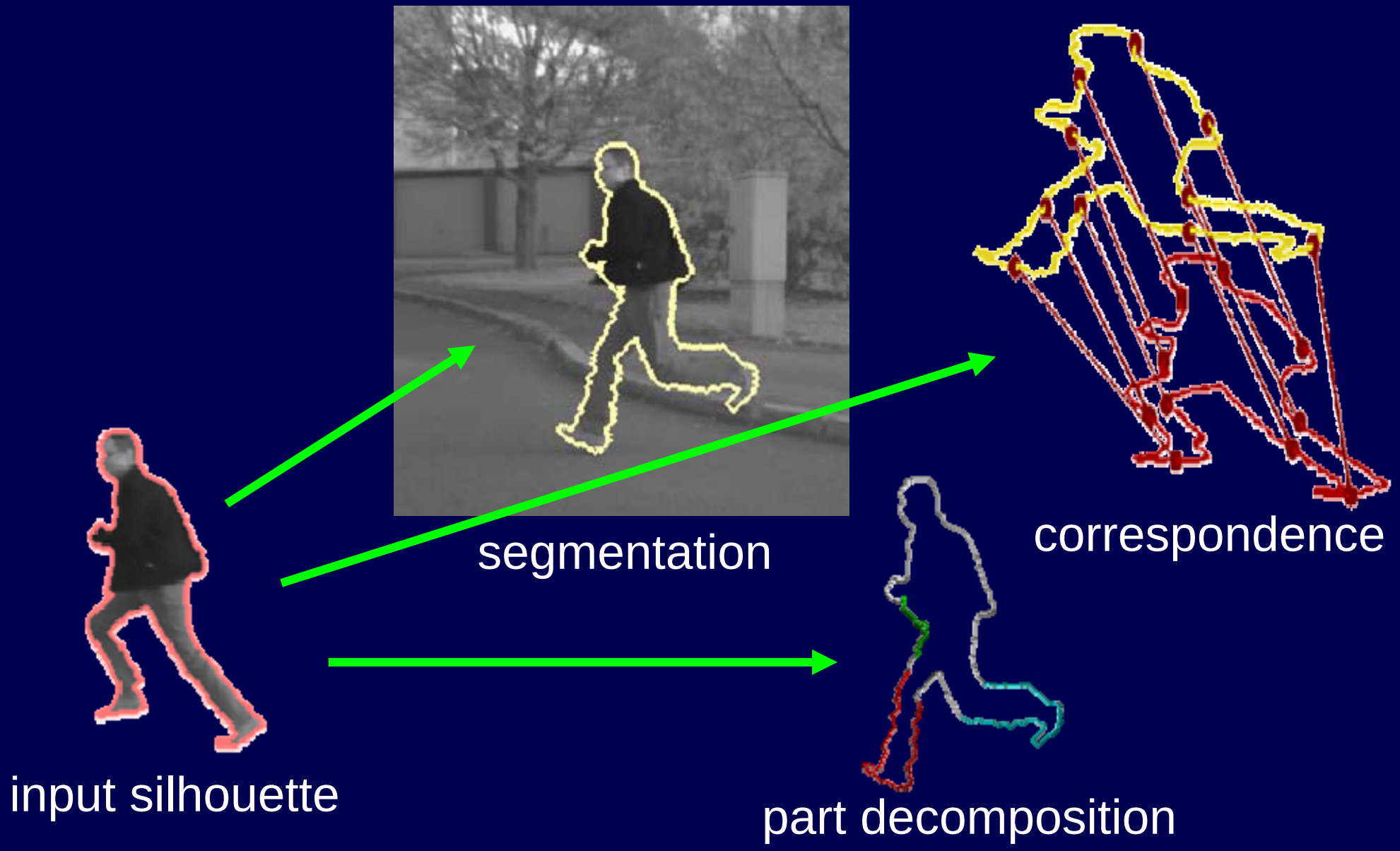
deformation

input silhouette



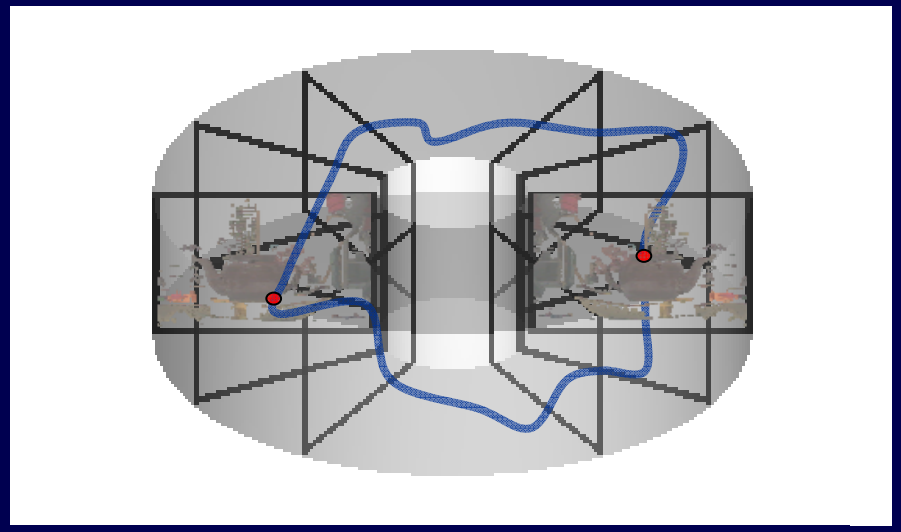
part decomposition

Segmentation, Correspondence Finding and Decomposition into Deformable Parts



Segmentation, Correspondence Finding and Decomposition into Deformable Parts

Summary



Matching as a shortest cyclic path

Segmentation & point correspondence

Optimal tracking in near realtime

Segmentation & part decomposition